**FB1: Find y’’ using implicit differentiation of**

The second derivative y’’ can be found by implicit differentiation, i.e., differentiating both sides of the equation twice. The first derivative is found to be

where both chain rule and difference rule is used on the left-hand side, while the right-hand side is found to be 0 because the derivative of a constant is always 0. Furthermore, the second derivative can be found using the product rule on the term and simple differentiation on like this:

Finally, rearranging for *y’’* and dividing both sides by 2 gives

**FB2: Prove by induction that for all .**

Let the statement P(n) be that for all . To prove a statement by induction, two steps have to be taken. Firstly, the base case has to be proven, which in this case is most easily done with because 1 is the smallest natural number. With the base case, the statement becomes

which results in a true statement. Secondly, the statement P(n) is assumed to be true. It can be restated as

The last step is to check whether P(n+1) can be proven by P(n). To do that one needs to keep in mind what the right-hand side for P(n+1) is supposed to be, which is . Therefore, the right-hand side of P(n) can be compared with a simplified version of this, which can be gained by comparing *n* and (*n*+1), and then taking them both to the *n*th power (which can be done without falling into fallacies because both sides are positive (*n* being a natural number)) like this:

This can then be combined with P(n) to make

from which it can be concluded that

When multiplying both sides by , which does not affect the inequality because it is a positive number, the statement becomes

This is the same as statement P(n+1), acquired from P(n).

Therefore, since both steps of induction (the base case for and the induction step from P(n) to P(n+1)) hold true, the statement P(n), or , is proven true by the mathematical principle of induction.

**FB3: Suppose the roots of the equation are and . Find the quadratic equation with roots and**

To find the sum and product of and , Vieta’s formulas can be used, which state that

and

Furthermore, Vieta’s formulas can also be used to find the coefficients of the new quadratic equation. The *x* term will be the negative value of the sum, which can be expressed as

And the constant term will be the product, which can be expressed as

When the sum and product of and are inserted into these equations, the sum evaluates to

and the product becomes

With these values, the new quadratic equation can be found, and it is

which can be simplified a bit by multiplying both sides by 6 and getting